Rotating cavity solitons in semiconductor microresonators

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(Received 21 July 2006; published 10 January 2007)

We predict the existence of a ring-like localized solutions in wide aperture passive semiconductor quantum well microresonators. These dark ring cavity solitons exist for positive cavity detuning in a frequency range where nonlinear dispersive effects prevail against absorptive ones. As a rule, destabilization of these ring solitons leads to the formation of radial asymmetric rotating solitons in externally driven optical cavities.

DOI: 10.1103/PhysRevE.75.017601 PACS number(s): 42.65.Tg, 42.70.Nq, 42.65.Pc, 42.55.Sa

Dissipative solitons (DSs) are self-consistent localized solutions of nonlinear, nonconservative systems with a substantial energy exchange with their environment [1]. Unlike conservative solitary waves, which form families, DS properties are completely determined by the system's parameters. Because of their robustness, DSs are nowadays regarded as key elements of future all-optical information processing schemes. A wide aperture nonlinear microresonator is a prominent example of an optical dissipative system [2–11]. Due to the internal feedback, it can adapt to the incident field in different ways, giving rise to multistability, patterns, and the formation of localized structures. It has been shown that roll and hexagonal patterns emerge due to modulational instability (MI) of plane wave (PW) solutions in a semiconductor microcavity driven by a coherent holding beam with and without carrier injections [3–8]. In a certain parameter domain, single constituents of this periodic pattern can be switched off separately [8]. Moreover, such a single object, usually termed cavity soliton (CS), can survive independently from the rest of the pattern, provided that an appropriate stable PW background exists. Cavity solitons are localized defects that either locally increase (bright CS) [4,9] or decrease (dark CSs) [5,8,10] the transmission of a Fabry-Perot cavity.

While stationary CSs correspond to a fixed point in the phase space, a limiting cycle, in general, describes a more complicated dynamical solution that changes its shape periodically in time. An example of a dynamically stable structure is the oscillating dark CS, which forms as a consequence of CS destabilization near a Hopf bifurcation [12]. Both the dark CS and its unstable linear mode are radial symmetric, resulting in oscillations of the entire structure without loosing its radial symmetry. On the other hand, an instability against asymmetric deformations may evoke a dynamical transformation of the localized structure into roll [6] or hexagon [7,11] patterns. The dynamics is more complicated if a linear mode does not exhibit radial symmetry (thus possessing a topological charge) and simultaneously undergoes a Hopf instability. As it was shown recently [13], the initially radial symmetric solution with or without topological charge may transform into a rotating CS. In the pertinent system, describing an active microresonator in the lasing regime, no external holding beam was required. In this case, rotation can be related to the structural topological charge because of the wave front vortices, although a rotating "two-hump" structure without initial phase dislocation was predicted as well.

Note that the rotation or motion of CSs is always associated with a curvature of the phase front. This phase curvature can be induced by the nonlinear phase shift of the CS's profile itself provided that a phase symmetry of the optical field exists. A prominent example of a system, where the solutions exhibit phase symmetry is the above-mentioned wide aperture semiconductor laser [13]. However, this symmetry will be broken in driven microresonators where the phase of the optical field is imposed by the external coherent holding beam. A homogeneous driving field with a flat phase prevents the occurrence of a self-consistent rotation. That is why, to the best of our knowledge, the existence of rotating CSs has not been reported yet for externally driven nonlinear microresonators. Note that an appropriate holding beam allows for CS steering. For example, a phase or amplitude gradient causes the CS to move [14], while a rotation takes place if the holding beam exhibits a doughnutlike shape [15].

The aim of this study is to show that destabilization of the internal linear mode of dark ring CSs lead as a rule to the formation of rotating CSs in a driven semiconductor microresonator, even if a homogenous holding beam is applied. Unlike droplets and dark ring CSs found in vectorial Kerr and degenerate optical parametric oscillator cases [16], ring CS considered here do not require equivalent stable states to exists.

The system we are considering is a wide aperture Fabry-Perot micoresonator endowed with semiconductor multiple quantum wells (MQWs) and driven by an external optical field (see Ref. [10]). For high-finesse resonators, only one longitudinal mode has to be taken into account and the mean-field approach can be applied to describe the light evolution. The optical nonlinearity is governed by the electron-photon interaction inside the MQWs where the photon has an energy slightly below the electronic band edge. Therefore, the standard equations for the intracavity field E and carrier density N evolution were used (see Refs. [4,8,10]).

$$\frac{\partial E}{\partial t} = -(1+i\theta)E + iD_{\mathrm{E}}\nabla_{\perp}^{2}E + E_{\mathrm{h}} - (i\xi' - \xi'')(N-1)E,$$

$$\frac{\partial N}{\partial t} = -\delta_{\mathrm{N}}N - \beta N^{2} + D_{\mathrm{N}}\nabla_{\perp}^{2}N - (N-1)|E|^{2}, \qquad (1)$$

where $\theta=\tau_{\mathrm{ph}}(\omega_{\mathrm{res}}-\omega_0)$ is the normalized cavity detuning from the resonance frequency ω_{res}; $\tau_{\mathrm{ph}}=2l/v(1-\rho_{\mathrm{u}}\rho_{\mathrm{l}})$ is the photon lifetime (1 is the effective cavity length, v is the group velocity of light, and $\rho_{u,l}$ are the reflectivities of the upper and lower mirrors). D_E and D_N are the diffraction and diffusion coefficients, and E_h is the holding beam amplitude. δ_N is the ratio of the photon lifetime in the resonator to the carrier recombination time, and β is the radiative recombination coefficient. In Eq. (1) the radiation-matter interaction is described by the linearized complex MQW susceptibility $(i\xi' - \xi'')(N-1)$, where the refractive part ξ' and the absorptive part ξ'' can be approximated by the refractive index and absorption spectra obtained experimentally for similar MQW structures (see Ref. [17]). For the operating wavelength (890 nm), the values are $\xi'=9.44$, $\xi''=0.81$, provided that the carrier density N is normalized to its transparency value ($N_0 \approx 0.8 \times 10^{18}$ cm^{-3}). Typical constants can be found, for example, in Ref. [4]. In normalized units used here, they amount to $D_E=6.53$, $\delta_N=5.8\times 10^{-4}$, $\beta=3\times 10^{-3}$, where the time is scaled to the empty cavity photon lifetime (of the order of $\tau_{ph} \approx 5$ ps), and the transverse coordinates x, y are scaled in 1 μm. The carrier generation-recombination dynamics is much slower than the photon dynamics in the resonator; therefore, the adiabatic elimination of the second equation for the carrier density must not be exploited, as in Ref. [12].

We assume a PW holding beam, which is justified if its width exceeds considerably the width of a typical CS (5–10 μm). Numerous theoretical and experimental investigations [1–11] confirm that bright and dark CSs can be found on a PW background. Here, we consider dark CSs that manifest themselves as a dark area in transmission on an otherwise bright homogeneous solution. The corresponding PW background is the upper branch of a required bistable input-output response curve of the wide aperture microresonator. It is known that conventional dark CSs are situated on a branch that bifurcates subcritically from either the limiting point of the upper PW branch or the point where MI sets in [3,8,10]. For the model (1), such conventional dark CSs have a large domain of existence for negative cavity detunings ($\theta<0$) in the absorptive/self-defocusing regime. This is the case if the photon energy is slightly below but close to the electronic band-gap energy. It means that photon absorption effects are larger or comparable to the refractive ones [in Eq. (1) $\xi'' > \xi'$ or $\xi'' \approx \xi'$].

Decreasing the operating frequency and, as a consequence, increasing the distance to the electronic band edge, refractive effects are the main contribution to the nonlinear dynamics ($\xi' \gg \xi''$). Thus, the high-power beam decreases the refractive index in the respective area. This self-defocusing regime is preferable for the formation of dark localized structures (see Refs. [1,2]). In this frequency range, different patterns and dark CSs were previously identified [5,12]. In the self-defocusing regime, the domain of existence of dark CSs extends over a positive detuning range ($\theta>0$) [in Fig. 1(a)]. Moreover, the dark CS exhibits a ring shape with a bright center [Fig. 1(b)]. Its circular form suggests the existence of a one-dimensional (1D) structure, which may stabilize itself in two dimensions (2D) due to curvature effects like the stabilization of the circular domain wall into the stable droplet [16]. However, there was no

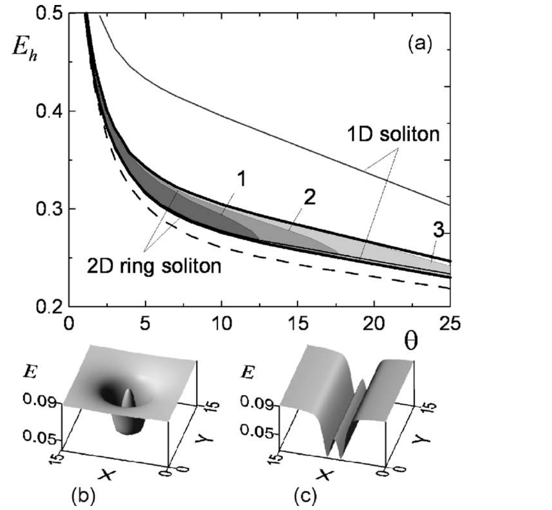

FIG. 1. (a) Domain of existence of a 2D ring CS (between thick solid lines) and a 1D dark CS (between thin solid lines) in parameter space given by detuning θ and amplitude of the holding beam E_h (normalized units). Shaded areas show the domain where dark ring CSs are unstable for $D_N=2.3\times 10^{-3}$ (1), $D_N=10^{-3}$ (2), $D_N=0$ (3). The dashed line describes the boundary of the PW bistability domain. Amplitude profiles (normalized units) of the stable 2D ring CS for $E_h=0.25$ (b) and 1D dark CS for $E_h=0.28$ (c), $\theta=20$, $D_N=2.3\times 10^{-3}$.

evidence of stable 1D dark CS with the canonical profile, i.e., a single intensity dip. Instead, it exhibits two minima and a bright center in transverse direction [Fig. 1(c)], resembling the cross-section of a 2D ring CS. Its existence domain is distinctly larger than the corresponding domain of dark ring CS [the area between thin lines in Fig. 1(a)]. These solitons are unstable against spatially modulated (y direction) perturbations except a small part of their existence domain.

The dark ring CSs bifurcate with zero amplitude from the limiting point of PW solutions [Fig. 2(a)], whereas their stable background may be related to the upper part of the bistable PW curve. Beyond the turning point of the CS branch (saddle-node bifurcation), they exist for smaller input intensity and become stable for a sufficiently large diffusion coefficient [curve 3 in Fig. 2(a)]. Thus, ring CSs are fundamental solitons that bifurcate subcritically from the PW solutions. We mention that higher-order structures (not considered here) are located at a curve that branches off from the succeeding turning point. The diffusion influences considerably the stability of dark ring CS. For example, the CSs branch, which is completely unstable for zero diffusion [line 1 in Fig. 2(a)], stabilizes if diffusion is accounted for [line 3 in Fig. 2(a)]. In general, the domain of soliton instability exists for small cavity detuning and shrinks with increasing diffusion coefficient [Fig. 1(a)]. Apparently the diffusion sweeps out any spatially modulated perturbations decreasing the growth rate of the unstable linear eigenmodes. We mention that the domain of ring CS existence decreases slightly with increasing diffusion [Fig. 2(a)].

To determine the stability of CSs, we performed a linear stability analysis of the stationary, radial symmetric solution. It reveals that various discrete linear modes (bound states) of the form $\delta\bar{U}_m(r)\exp(im\varphi)$ exist, which either grow or decay

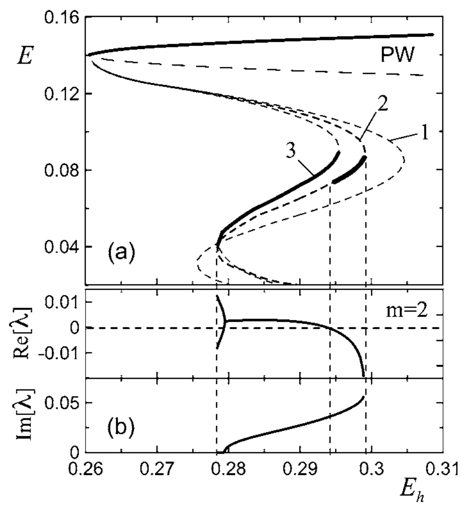

FIG. 2. (a) Minimum amplitude (dip) of the ring CS (solid lines: stable solution; dashed lines: unstable ones) vs amplitude of the holding beam for $\theta=10$ and different diffusion coefficients: $D_N=0$ (1); $D_N=2.3\times10^{-3}$ (2); $D_N=5\times10^{-3}$ (3). (b) Real and imaginary parts of eigenvalue λ of the mode with topological charge $m=2$ for the ring CS (2).

exponentially as $\exp(\lambda t)$. Here, we introduced the field and carrier density profile of the linear eigenmode as $\delta\bar{U}_m(r)=\{\delta E_m(r),\delta N_m(r)\}$ around the stationary CS solution $\bar{U}_0(r)$. Instability occurs if any eigenvalue λ has a positive real part. Ring CSs are unstable on the upper branch because of a growing, radial symmetric ($m=0$) linear mode. It has a real-valued eigenvalue that passes zero at the turning point as expected. The eigenvalue of another leading mode is complex valued. Near the turning point, its real part is negative (stable stationary CS) but crosses zero for smaller input intensities (unstable stationary CS); see Fig. 2(b). Moreover, this unstable eigenmode has a radially asymmetric shape with topological charge $m=2$ that leads to a rotation (with frequency $\Omega=\mathrm{Im}[\lambda]/m$) rather than a CS oscillation appearing slightly below Hopf bifurcation. The imaginary part ($\mathrm{Im}[\lambda]$) decreases with decreasing holding beam amplitude (E_h) down to a critical point where two complex conjugate eigenvalues fuse and two real-valued ones emanate [see $E_h\approx0.279$ in Fig. 2(b)]. Note that there is no instability related to complex eigenvalues for a PW solution if the carrier recombination time is either much larger (as in Ref. [4]) or much smaller (as in Ref. [12]) than the photon lifetime.

The linear analysis elucidates the dynamics of stationary solutions only for small perturbations close to the bifurcation points. To get a complete dynamical picture, we performed the direct numerical integration of Eqs. (1), applying a random noise to the unstable ring CS as an initial condition. First, the unstable ring CS loses its radial symmetry and starts oscillating [Fig. 3(a)]. This behavior can be explained by taking into account that two modes with complex conjugated eigenvalues exist, which correspond to opposite rotational directions. They grow independently from the random noise with approximately equal amplitudes $\exp(\mathrm{Re}[\lambda]t)$. Their superposition results in a "standing wave," i.e., temporal oscillations with the frequency $\omega=\mathrm{Im}[\lambda]$. These

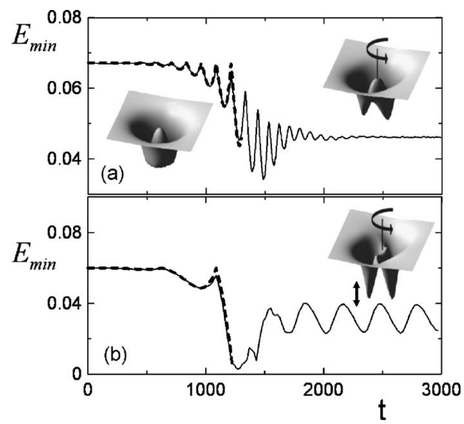

FIG. 3. Temporal dynamics (time is expressed in τ_{ph}) of the minimum amplitude (dip) of the unstable ring CS for different diffusion coefficients (a) $D_N=1.5\times10^{-3}$; (b) $D_N=0$. The unstable ring CS (left inset) transforms into a rotating CS [right inset in (a)] or rotating-oscillating CS [inset in (b)]. The thick dashed lines are results of the linear stability analysis. Parameters: $\theta=10$, $E_h=0.293$.

predictions of the linear stability analysis describe perfectly the oscillation dynamics at the beginning of the destabilization process (thick dashed lines in Fig. 3). Further simulations shows that nonlinearity leads to the deformation of both unstable eigenmodes, and the energy exchange between them gives rise to the eventual stable rotation of CS [right inset in Fig. 3(a)]. The direction of rotation is determined by the initial perturbation of the unstable ring CS, whereas the rotation frequency is about $\Omega=\mathrm{Im}\,[\lambda]/2$, provided that the parameters are close to the bifurcation point. Otherwise, the rotation can be accompanied by additional oscillations, as it

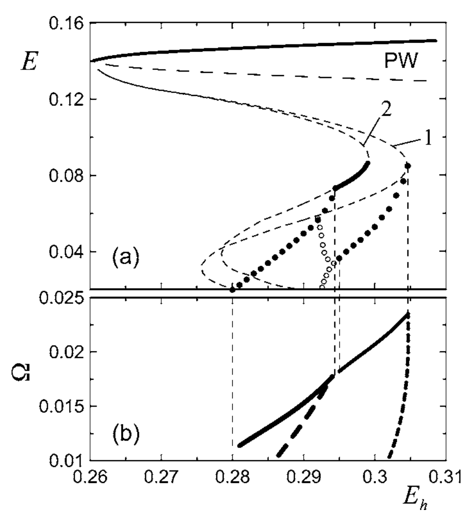

FIG. 4. (a) Minimum amplitude of the rotating CS (filled circles) vs amplitude of the holding beam for $\theta=10$ and different diffusion coefficients: $D_N=0$ (1); $D_N=2.3\times10^{-3}$ (2). Open circles denote the branches of the rotating-oscillating CSs. (b) Rotation frequency $\Omega=2\pi/T_0$ (T_0 is the period of rotation expressed in τ_{ph}) of CS vs holding beam amplitude E_h. Solid lines represent results of the direct numerical calculation of Eq. (1), whereas dashed lines are the results of linear stability analysis ($\Omega=\mathrm{Im}[\lambda]/2$).

is illustrated in Fig. 3(b). Further changes of parameters result in a chaotic oscillations of CSs and their eventual decay.

The dips of the rotating CS belong to the branch that emanates supercritically from the stationary radial symmetric solution at Hopf bifurcation [filled circles in Fig. 4(a)]. A special situation appears if the limit point of CSs and the Hopf bifurcation coincide [line 1 Fig. 4(a)]. Then the real parts of both eigenvalues belonging to eigenmodes with different topological charges (namely, radial symmetric $m=0$ and another with $m=2$) become simultaneously zero. This coincidence of two types of instabilities, Turing and Hopf, was previously found in optical parametric oscillators with saturable losses [18] where time-periodic patterns emerge as a consequence of secondary bifurcation. In our case, the dynamically stable rotating CS emanates from the completely unstable branch of stationary solutions in the turning point. The rotating CS is a stable radial asymmetric localized structure in a reference frame rotating with a definite frequency. However, it experiences a secondary bifurcation and starts to oscillate for a smaller holding beam amplitude [open circles in Figs. 4(a) and 3(b)].

According to the results of the linear stability analysis, the imaginary part of the growth rate of the unstable eigenmode increases with increasing holding beam amplitude [Fig. 2(b)]. As a consequence, the rotational frequency should behave similarly ($\Omega=\mathrm{Im}[\lambda]/2$) provided that the rotating CS shape does not deviate too much from the ring CS. Direct numerical simulations of Eq. (1) show that this statement is valid near the bifurcation point [Fig. 4(b)].

In conclusion, we have identified fundamental dark ring CS for positive detuning in wide aperture semiconductor microresonator driven by an external holding beam. The linear stability analysis and extensive numerical simulations show that close to Hopf bifurcation these ring CSs transform into rotating CSs, even if a PW holding beam is applied. Moreover the branch of stable rotating CSs can bifurcate from a completely unstable branch of ring CSs.

We are grateful to D. Michaelis and C. Etrich for helpful discussions on the stability of dark CSs in semiconductor microcavities.

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